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FACTORS AFFECTING SPACE EFFICIENCY
OF PALLETIZED STORAGE

A THESIS

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FACTORS AFFECTING SPACE EFFICIENCY

OF PALLETIZED STORAGE

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FOREWORD

The subject of this study is the result of several years experience by the author in developing practical warehouse layouts in the retail grocery industry. The varying claims of fork lift truck manufacturers and general disagreement among operators has led the author in search of a more exact measure of the factors which affect the efficiency and the economy of space.

The encouragement and guidance, as thesis advisor, of Dr. J. J. Moder is gratefully acknowledged. At several critical stages of the work, Dr. Moder supplied important advice in developing the equations and in reducing them to meaningful figures. Acknowledgement is also given for the advice and council of Dr. James H. Armstrong.

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SUMMARY

Economy of storage space and efficiency of the pallet placement operation have often been considered in conflict with each other. Many considerations enter into the economy of pallet placement, such as size of pallets, size of fork lift truck, width of aisle, column spacing, and type of operation. The "slant" angle of the pallets, with respect to the aisle, and the best spacing between adjacent pallets will vary with each set of conditions. No single angle or spacing will fit every combination of factors or satisfy all of the many conditions which enter into a given warehousing problem.

In this study mathematical models were developed and manipulated to determine optimum values of slant angle and pallet spacing. The measure of effectiveness chosen for determining optimality was the ratio of the area of the pallet or pallets facing an aisle to the total relevant warehouse area required to house and service the pallets, assuming the aisle services two rows of pallets. From fork lift truck specifications supplied by the manufacturers, an appropriate range of values for length and width of the fork truck, turning radius, and maneuvering characteristics were selected. Several popular sizes of pallets in general use in the industry were also selected.

Three model equations were written to describe the efficiencies of the following cases:

Case I. Slant angle, zero degrees; pallet spacing zero.

Case II. Slant angle, zero degrees; pallet spacing not zero.

Case III. Slant angle, not zero degrees; pallet spacing zero.

A solution to each of the equations in terms of the pallet dimensions, fork truck characteristics, slant angle and pallet spacing are presented in tabular form showing the effect of departures from the optimum values.

The results of the study indicate that the optimum space between pallets is zero, the optimum pallet placement is perpendicular to the aisle, and that pallet length and number of pallets in depth should be made as large as possible. The parameters, pallet width and number of pallets in width had little effect on the resulting efficiencies. It was also noted that the sum of the parameters, turning radius, operating clearance, and distance from face of forks to center of front axle, should be as small as practical.

CHAPTER I

INTRODUCTION

Warehouse Layout.--While the ideal procedure when making a plant layout is to first establish the handling system and then build the plant around it, ordinarily the plant is already built and merchandise and assembly lines must be spotted within the confines of existing walls and columns. This complicates the problem and often requires compromising the optimum layout. Yet, if the same techniques which are used when designing new facilities were used to plan the layout for old plants, many of these problems could be minimized, handling costs could be lowered, and production rates could be increased.

Prior to the introduction of the hand and power fork lift truck warehoused merchandise was placed into storage manually. This required the handling of each case of merchandise as it was placed into storage and an additional handling of the same case when it was withdrawn from storage. One case was placed on top of another, one at a time, and was withdrawn in the reverse manner. If ceiling height and floor load capacity permitted, they were stacked as high as the warehouseman could reach either from the floor or by climbing on top of the merchandise. It could also be block-stacked with no space wasted between merchandise.

With the introduction of the manual and, later, the power fork lift truck, it was found that unit loads of merchandise could be moved from point of receipt to storage in less time and at a lower cost. However, the addition of these new handling devices created additional problems in the requirement for more aisle space and clearance between adjacent stacks in order to maneuver. Consequently, the ratio of storage space to total space was decreased. In an effort to recapture this loss in storage area, unit loads were placed on top of one another as high as the merchandise would stand without toppling or crushing.

While most authors recognize the importance of conserving space very few have treated it quantitatively.

Literature Search.--A thorough search of the literature on the efficient use of warehouse space failed to turn up any reference to a quantitative study of the subject. This has been essentially an empirical field.

The following material quoted from Sidney Reibel (1) presents an excellent picture of the important part played by aisles in the plant layout.

> In the average industrial plant, aisles are the highways and byways, the roads, on which almost all movement of workers and materials takes place .. with the exception of course of the work accomplished by conveyors and cranes. Aisles take up a lot of space, sometimes so much that plant efficiency is affected. Therefore, careful consideration of the location and arrangement of aisles is not only advisable but is likely to be profitable as well. On the other hand, carelessness and neglect of aisles may prove to be costly.
>
> One of the principal factors in setting minimum widths for interior trucking aisles is the space necessary for maneuvering power lift trucks. Turning radius, and .. as well as side .. clearances, and size of loads of specific equipment must be checked by clearance diagrams to help determine the width of aisles.

Additional information concerning the important role played by aisles and floor area may be found in the text by James H. Apple (2):

> Aisles are non-productive areas and each square foot used for aisles is lost to production. Production pays for its floor area .. aisles do not.
>
> Each square foot of floor area in a plant costs money. One manufacturer, for example, has calculated his floor area cost to be $1.00 per square foot per month. This amount includes all overhead costs. Only if each square foot is used to best advantage can the attending overhead costs per unit of product be kept down. Floor area occupied by equipment in operation pays its own way. Unoccupied, wasted or idle floor area is a burden on the rest of the plant.

From these statements and others that equally stress the importance of conserving floor space as well as aisle width, it is reasonable to assume that any effort made in the direction of utilizing existing space more efficiently or planning new space for optimum use will minimize the capital investment.

Shubin and Madehelm (3) in their text mention the importance of storage; however, they too provide no clue as to how this problem can be approached on a mathematical basis. The following is an excerpt from the text:

> Storage activities provide the following services to the plant; to receive all materials and supplies; to protect and reduce wastage of materials due to deterioration, theft, and breakage; upon authorization to issue materials in the required manner and quantities, and at the specified time; and to control temporary storage of work in process.
>
> Areas should be arranged in the floor plan to provide the maximum storage service at minimum cost. A good stores layout provides the following benefits: (1) efficient utilization of floor space devoted to storage, (2) quick availability of materials for manufacturing, (3) aids material control by facilitating physical turnover of materials and ease of taking physical inventory, (4) provides maximum flexibility of storage arrangement to permit changes and expansion of inventory at low cost.
>
> The ratio of aisle space to the total storage area should be as low as is practical.

Some work has been done toward providing an equation relating aisle width to the desired dimensions of pallet size and fork lift truck specifications. Stocker (4) shows the following in his text:

$$A = TR + X + L + C$$

where width (W) is less than 2B, and

$$A = TR + \sqrt{(X + L)^2 + (\frac{W}{2} - B)^2} + C$$

where width (W) is more than 2B and where

A = Aisle width.

B = Distance from center line of truck to center line of point about which truck turns when steering wheels are in extreme cramped position

TR = Turning radius.

L = Length of load.

W = Width of load.

C = Clearance set at 6 inches.

X = Distance from center line of drive axle to face of forks.

While this equation will assist in determining the aisle width, it in no way relates it to storage area. (Further investigation reveals that this equation is valid only for the condition of $c = 0$, $\alpha = 0$, as defined in Chapter II of this thesis.)

There is further mention by Stocker (4) of other methods of stacking merchandise:

> When material is stacked at 90° to the aisle, the aisle must be of sufficient width to allow for the overall length of the fork truck, plus the load.

> For large center sections in which minimum aisle widths are desired, the 45° angle method should be given consideration. Narrower aisles can be used with 45° angle method since the equipment makes only a quarter turn to face the point of storage. When pallets are stored in a center section, 45° stacking causes some loss of space.
>
> When part of palleted material in storage is to be withdrawn at frequent periods, or if one or more packages are to be removed from pallets, the 45° method of storing proves very advantageous. The loss of space caused by partial withdrawal of lots in storage leaves space which cannot be filled until the remainder of the material is moved. This loss of space through partial delivery of lots is called "honey combing." However, by the 45° method, short rows are provided easily so that more effective utilization is made of the available space.

A later search of literature written on the subject of warehouse storage produced a technique in determining how the storage facility should be planned. The text by Haynes (6) has the following paragraph dealing with the subject:

> Many who are planning a storage facility feel that much space can be saved by using 45-deg instead of the more conventional 90-deg piling. The fact that frequently is overlooked is that when storage space is set up on the former basis, more space is required in arranging the stored pallets at a slant than when they are perpendicular to the aisle - -. The safest procedure in planning a new layout is to make a scale drawing of the proposed storage area and then, with templates made to the same scale, test the different possibilities.

Further search of literature available on the subject of storage space utilization reveals only that Stocker (4) and Barker (5) agree that the 48" x 48" pallet is the most practical size. This is not the generally accepted size in use in the industry. The 48" x 40" pallet appears more frequently than any other size, although there is still wide disagreement as to the proper size to use. It would appear practical to pursue further the relationship of pallet sizes as they affect optimum storage space utilization in an effort to establish equations with which one can base the layout of storage space in an optimum manner. It also is apparent that calculations should be made of the efficiency of slant placement of pallets and placement with lateral clearance between pallets under as wide a range of conditions as is practical. It would appear important to have available the tabulated results of these investigations so that any person responsible for plant layout can choose the optimum values of pallet and fork lift truck to fit the conditions of existing buildings or determine the minimum capital to be invested in new buildings by choosing optimum values for the required equipment.

CHAPTER II

THE MODEL EQUATIONS

Description of the Process.--Plant layout embraces the physical arrangement of industrial facilities. This arrangement, either installed or in plan, includes the spaces needed for material movement, storage, indirect laborers, and all other supporting activities or services, as well as for operating equipment and personnel. The term "plant layout" sometimes means the existing arrangement, sometimes proposed new layout plan, and often the area of study or the work of making a plant layout. Hence, layout may be an actual installation, a plan, or a job.

One of the most important factors in plant layout is that of material handling. Material handling consists of moving packages of a given size and weight, and transportation of these packages from one location to another. This operation may be repeated many times with many variations. Material handling essentially involves these two operations: (1) storage, and (2) transportation. Let us consider here the problem of storage.

Although not strictly material handling, storage is so closely related to it that it should be considered here. In general, storage can be classified as one of two types: (1) temporary - considered to be the temporary holding of material in any location due to inability to utilize it immediately, the duration is usually short; or (2) permanent - considered to be a planned process for storing materials, usually in a predetermined location, for the purpose of controlling the supply or to render immobile materials not required for immediate use. The time interval of storage is usually long. Let us consider a storage area of width X, and depth Y, into which we wish to place as many pallets as possible consistent with ease of handling. For access to the storage area, we will consider an aisle of width D, along which will travel and maneuver a self-propelled fork lift truck. Merchandise to be placed in the storage area will be picked up by the fork lift truck after it has been placed on the pallet and then be transported along the aisle to the desired location in the storage area. The fork lift truck will then be maneuvered to "slant" angle of α degrees from the axis of the aisle toward the storage area and the pallet will be deposited in the storage area. The process of storing the pallets will be repeated until the entire area is filled.

In this study the following notation will be used:

a = width of pallet.

b = length of pallet.

c = lateral clearance between pallets.

d = operating clearance in aisle width to maneuver fork lift truck, (6").

D = width of aisle.

E = ratio of the pallet area to the total relevant area, expressed as a percentage.

g = distance from face of forks to center line of the front axle of the fork lift truck.

K = the sum of g, d, and R.

L = distance from leading edge of the aisle to the center of the point about which the fork truck front wheel turns.

N_d = number of pallets in depth from the aisle.

N_w = number of pallets facing the aisle.

n = total number of additional pallets which may be placed in the corners of the storage area when pallets are angle stored (i.e. when $\alpha \neq 0$).

p = the distance from the center line of the fork lift truck front axle to the end of the pallet.

r = turning radius of fork truck front wheels.

R = turning radius of rear of fork lift truck.

R_1 = turning radius to end of pallet.

S = the distance the pallet must be moved from the storage area before it clears the adjacent pallet for turning (see Fig. 2).

w = width of the fork lift truck.

X = length of storage area of N_w pallets facing the aisle.

Y = depth of storage area of N_d pallets from the aisle plus one-half the aisle width.

α = angle of rotation of the pallet with respect to the center line of the aisle.

β = the sum of the angles σ and θ_f (see Fig. 2).

μ = the angle formed by the side of the pallet and the fork truck turning radius (see Fig. 2).

σ = the angle formed by the side of the pallet and the pallet turning radius (see Fig. 2).

θ_o = the initial angle formed by the side of the pallet being rotated and the side of the adjacent pallet (see Fig. 2).

θ_f = the final angle formed by the pallet being rotated and the side of the adjacent pallet at the moment complete freedom of rotation is achieved (see Fig. 2).

In this study, the following three cases will be studied:

Case I. $c = 0, \alpha = 0$.

Case II. $c \neq 0, \alpha = 0$.

Case III. $c = 0, \alpha \neq 0$.

<u>Case I. $c = 0, \alpha = 0$.</u>--Referring to Figure 1, the reader will note that the restrictions placed on the movement of the pallet, by assuming no clearance between the pallet to be moved and the adjacent pallet, precludes any rotation of the fork lift truck and pallet until the end of the pallet is withdrawn completely from the storage area into the aisle. The end of the pallet being withdrawn must clear the face of the adjacent pallet before there is complete freedom of rotation for the pallet and fork truck in order that the fork truck can travel along the aisle.

Since the fork truck and the pallet must move at right angles to the axis of the aisle and the end of the pallet being withdrawn from storage must clear the leading edge of the aisle before any rotation may be accomplished, we may write the equation for the aisle width as a sum of pallet and fork truck parameters

$$D = b + g + R + d \qquad (1)$$

valid for, $a \leq D - d$.

Referring again to Figure 1, we see that the storage area in which the pallets rest is composed of N pallets and since no clearance is assumed between pallets, the area utilized is equal to the area occupied by the N pallets. This may be expressed as

$$\text{AREA UTILIZED} = a\, b\, N_w\, N_d \ .$$

Assuming each aisle is shared by two rows of pallets, the total relevant area is characterized as the area occupied by the pallets plus one-half the area of the aisle on which the pallets face. This may be

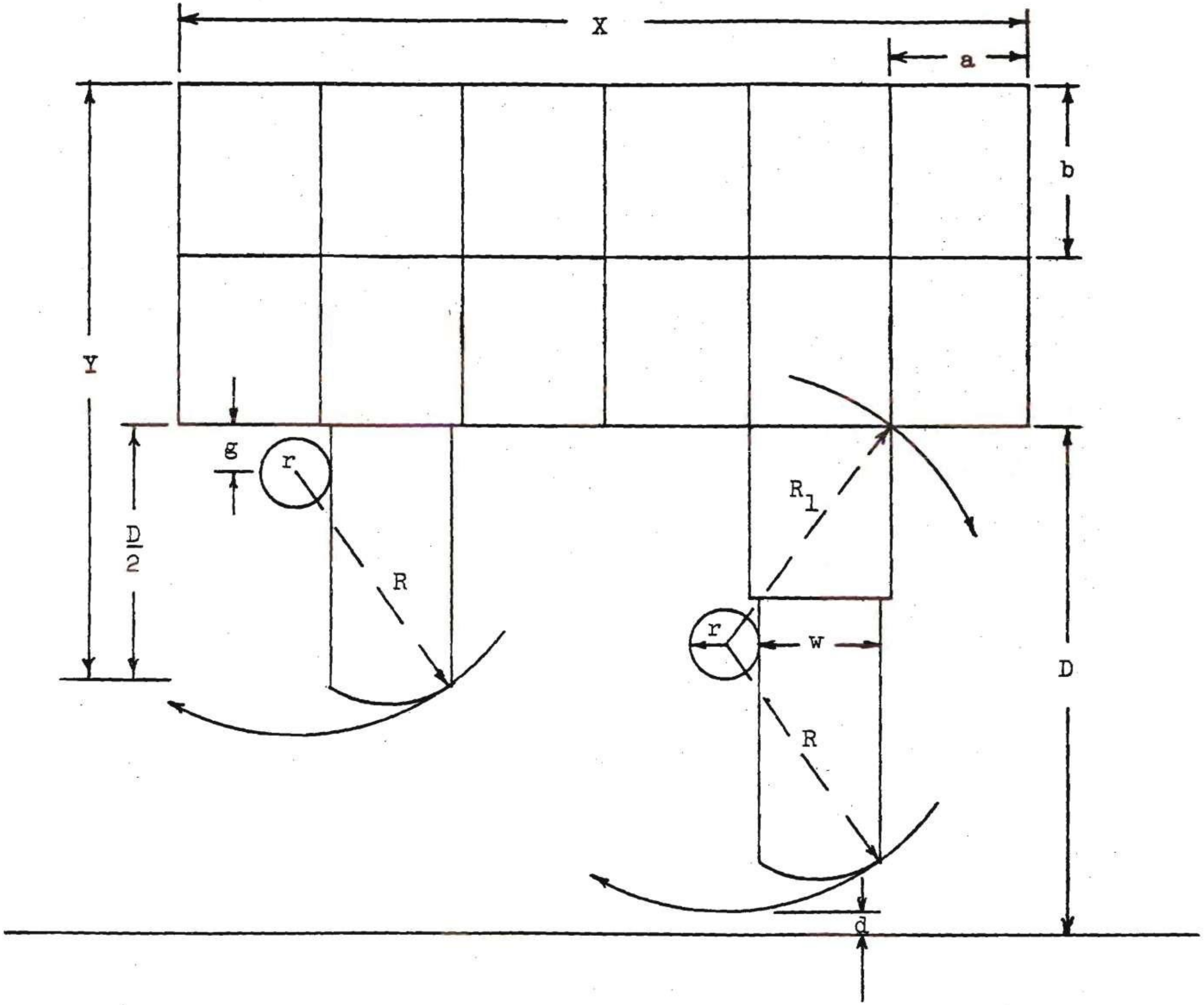

Figure 1. Right Angle Placement With No Clearance Between Pallets.

expressed as

$$\text{TOTAL AREA} = N_w(a + \frac{c}{2})\ (N_d b + \frac{D}{2})$$

$$= N_w(a + \frac{c}{2})\ (N_d b + \frac{b + g + R + d}{2})\ .$$

The equation for the efficiency of utilization of the space may be expressed as

$$E = \frac{\text{AREA UTILIZED}}{\text{TOTAL AREA}} \times 100\% \ , \text{ and}$$

$$E = \frac{100\ a\ b\ N_w N_d}{N_w\ (a + \frac{c}{2})\ (N_d b + \frac{b + g + R + d}{2})}\ \%$$

$$E = \frac{400\ a\ b\ N_w N_d}{N_w\ [2a + c]\ [(2N_d + 1)\ b + g + R + d]}\ \%\ .$$

Now let $c = 0$ as assumed in Case I, and cancel the common factors ($2\ a\ N_w$) to give

$$E = \frac{200\ N_d\ b}{(2N_d + 1)\ b + (g + R + d)}\ \%\ . \tag{2}$$

Examination of Equation (2) indicates the following:

(i) E is independent of a and N_w.

(ii) E increases monotonically as b and N_d increase.

(iii) E decreases monotonically as the fork truck parameter ($g + R + d$) increases.

Case II. $c \neq 0,\ \alpha = 0$.--Let us examine the movement of the fork lift truck and pallet as the pallet is withdrawn from the storage area. If the reader will refer to Figure 2, position 1, it will be noted that there is no immediate restriction to the rotation of the pallet being withdrawn. The fork truck may pivot about the center of the turning radius of the front wheel until the corner of the pallet being withdrawn reaches the side of the adjacent pallet. There is no rearward movement of the fork truck or pallet up to this moment. In this rotated position, the pallet being moved and the adjacent pallet form an angle between the two sides which will be denoted by θ_o. The distance between the two pallets into which the pallet being moved has been rotated is designated by c. The distance from the point of contact of the rotated pallet and the stationary pallet to the face of the stationary pallet is designated by S.

Assume for convenience that the corner of the movable pallet, which contacts the stationary pallet, moves normal to the aisle a distance S, at a uniform velocity V. The time of travel t, is then

$$t = S/V \ .$$

Since the truck is a rigid body and can not move into the space occupied by the adjacent pallet, the other end of the truck must move away from the adjacent pallet so the velocity vector, $V \sin \theta$, is acting in this direction. This will be noted in Figure 2, position 2, and can be described by the following differential equation

$$V \sin \theta = p \frac{d\theta}{dt} \ .$$

Then we can write

$$\int \frac{V}{p} \, dt = \int \frac{d\theta}{\sin \theta} \ ,$$

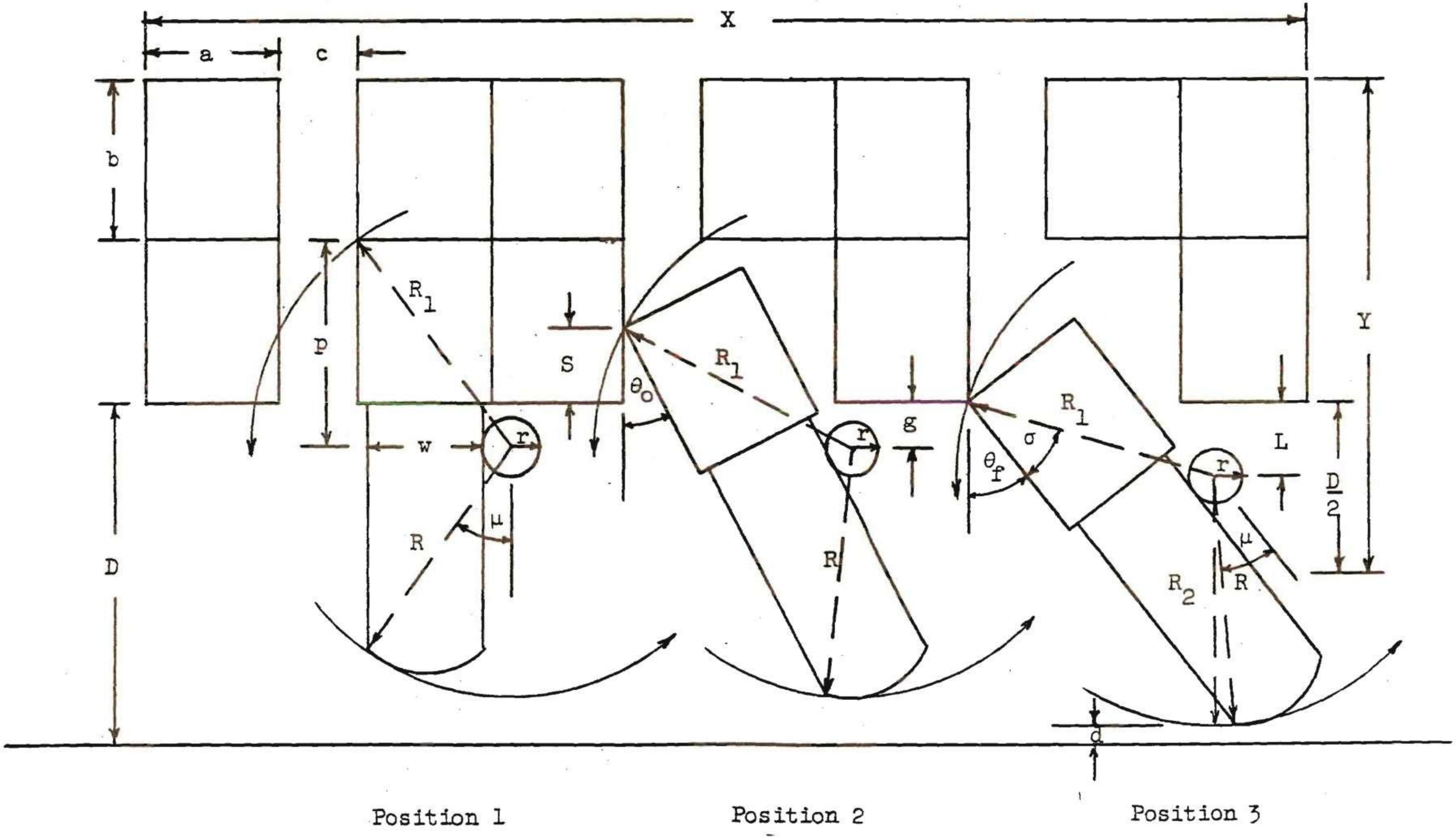

Figure 2. Right Angle Placement with Lateral Clearance Between Each Second Pallet.

a solution of which is given by

$$\ln(\csc\theta - \cot\theta) = \frac{V}{p}\, t + c_1,$$

where c_1 is an arbitrary constant. Since $\csc\theta - \cot\theta = \tan\frac{\theta}{2}$ we can write this as

$$\ln\tan\frac{\theta}{2} = \frac{V}{p}\, t + c_1 \ .$$

The constant c_1 can be evaluated from the boundary condition $\theta = \theta_o$ when $t = 0$, giving the following for c_1.

$$\ln\tan\frac{\theta_o}{2} = c_1 \ .$$

Thus

$$\ln\tan\frac{\theta}{2} = \frac{V}{p}\, t + \ln\tan\frac{\theta_o}{2}\ , \quad \text{or}$$

$$\ln\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_o}{2}}\right) = \frac{V}{p}\, t \ .$$

Now, when $t = S/V$, we let $\theta = \theta_f$, which reduces to

$$\frac{\tan\frac{\theta_f}{2}}{\tan\frac{\theta_o}{2}} = e^{S/p}\ , \quad \tan\frac{\theta_f}{2} = \tan\frac{\theta_c}{2}\, e^{S/p}, \text{ and finally}$$

$$\theta_f = 2\arctan\left(\tan\frac{\theta_o}{2}\, e^{S/p}\right) . \qquad (3)$$

In equation (3), θ_f is the final angle between the side of the pallet being moved and the side of the adjacent pallet at the moment complete freedom of rotation is achieved. This position will be noted in Figure 2, position 3. It will be shown below that, given θ_f, calculation of the aisle width follows in a straight forward manner. To this end, let us express the constants θ_o, S, and p, appearing in equation (3), in terms of the constants already defined for the pallet size, truck characteristics, and lateral clearance between pallets.

Inspection of Figure 2 indicates that

$$p = b + g \tag{4}$$

and Position 2 of the Figure shows that

$$(S + g)^2 = R_1^2 - \left(\frac{w + a}{2} + c + r\right)^2 ,$$

or

$$S = \sqrt{R_1^2 - \left(\frac{w + a}{2} + c + r\right)^2} - g \tag{5}$$

We may also see from Position 1 of Figure 2, that

$$R_1^2 = \left(r + \frac{w + a}{2}\right)^2 + (b + g)^2 \tag{6}$$

Substitution of equation (6) in equation (5) changes the latter as follows

$$S = \sqrt{\left(r + \frac{w + a}{2}\right)^2 + (b + g)^2 - \left(\frac{w + a}{2} + c + r\right)^2} - g \tag{7}$$

The use of the expression $\frac{w + a}{2}$ in the preceding equations can best be described by referring to Figure 3, where the conditions of $a > w$

and a < w are illustrated. Since the parameter r is a function of the fork lift truck and not of the pallet, it follows that any values assigned to this parameter must be measured from some base point on the fork truck. This base point for the front wheel turning radius is measured from the outside edge of the front wheels of the fork truck. Now, in order to calculate the value for R_1, when a > w, we must construct the right triangle whose hypotenuse is R_1, and whose sides are (b + g) and $(\frac{a}{2} + \frac{w}{2} + r)$ respectively. Upon inspection of Figure 3 for the condition of a < w, the reader will see that the equations for both conditions are the same and therefore the expression $\frac{w + a}{2}$ is valid for a < w or a > w.

The equation for θ_o can be written as follows

$$\theta_o = \text{arc sin} \left(\frac{w + a + 2c + 2r}{2R_1} \right) - \text{arc sin} \left(\frac{w + a + 2r}{2R_1} \right) \tag{8}$$

Substitution of equation (6) in equation (8) yields

$$\theta_o = \text{arc sin} \left(\frac{w + a + 2c + 2r}{\sqrt{(r + \frac{w + a}{2})^2 + (b + g)^2}} \right) - \text{arc sin} \left(\frac{w + a + 2r}{\sqrt{(r + \frac{w + a}{2})^2 + (b + g)^2}} \right) \tag{9}$$

Finally, substitution of equations (4), (7), and (9) in equation (3), yields

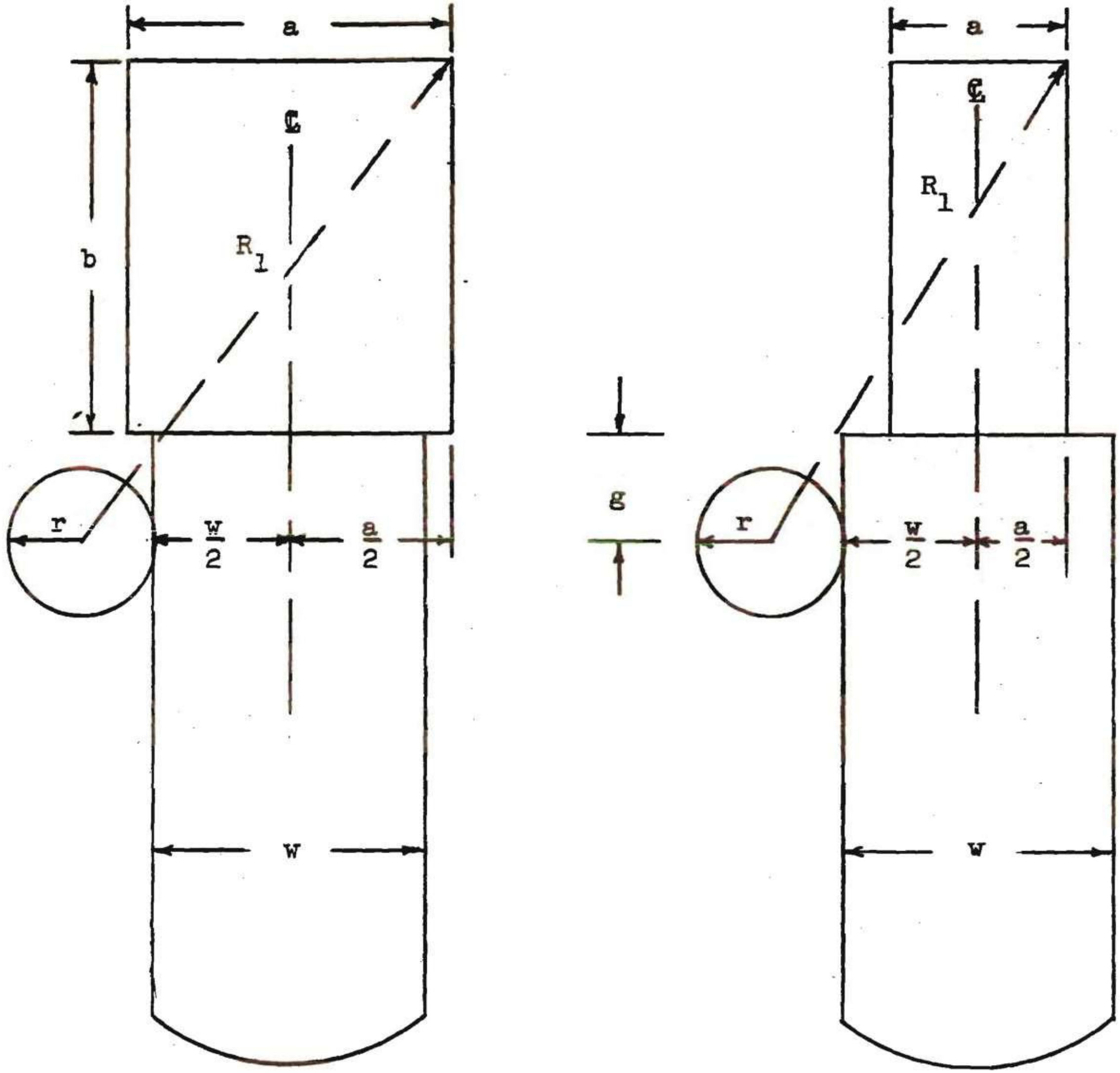

Figure 3. Transitions Valid for $w \leq a \leq w$.

$$\theta_f = 2 \text{ arc tan } \left[\tan \frac{1}{2} \left\{ \text{arc sin} \left(\frac{w + a + 2c + 2r}{\sqrt{(r + \frac{w + a}{2})^2 + (b + g)^2}} \right) \right. \right.$$

$$\left. - \text{ arc sin} \left(\frac{w + a + 2r}{\sqrt{(r + \frac{w + a}{2})^2 + (b + g)^2}} \right) \right\}$$

$$\left. \exp \left\{ \frac{\sqrt{(r + \frac{w + a}{2})^2 + (b + g)^2 - (r + \frac{w + a}{2}) + c)^2} - g}{b + g} \right\} \right] \quad (10)$$

Consider now the effect of $c \neq 0$ on the aisle width D. Referring again to Figure 2, we can write

$$D = R_1 \cos (\theta_f + \sigma) + R_2 + d, \quad (11)$$

where

$$R_2 = R; \quad \theta_f \leq \mu, \quad (12)$$

or

$$R_2 = R \cos (\theta_f - \mu); \quad \theta_f \geq \mu . \quad (13)$$

The angle σ is a function of the pallet and fork truck parameters and the angle μ is a specific function of the fork truck; they can be expressed as follows:

$$\sigma = \text{arc sin} \left(\frac{w + a + 2r}{2R_1} \right), \quad (14)$$

and

$$\mu = \text{arc sin} \left(\frac{w + r}{R} \right) . \quad (15)$$

Substitution of equation (12) in equation (11) yields the equation for the aisle width in terms of the constants already defined and can be written as:

$$D = R_1 \cos(\theta_f + \sigma) + R + d \; ; \quad \theta_f \leq \mu \, . \tag{16}$$

In like manner, by substituting equation (13) in equation (11), the following result is obtained:

$$D = R_1 \cos(\theta_f + \sigma) + R \cos(\theta_f - \mu) + d \; ; \; \theta_f \geq \mu \, . \tag{17}$$

Having developed equations for the aisle width D in terms of known parameters, we can now write the expression for the efficiency of utilization of floor space as follows:

$$\text{AREA UTILIZED} = a \, b \, N_w N_d$$

$$\text{TOTAL AREA} = N_w \left(a + \frac{c}{2}\right) \left(N_d \, b + \frac{D}{2}\right)$$

$$E = \frac{400 \, a \, b \, N_d}{(2a + c)(2N_d \, b + D)} \, \% \tag{18}$$

where the common factor N_w has been cancelled, and the quantity D is given in equations (16) or (17). Clearly, in this case, the efficiency is a maximum when c = 0, and decreases monotonically as c increases; also E is independent of N_w as in Case I.

<u>CASE III. $c = 0, \alpha \neq 0$.</u>--It will be evident to the reader, when referring to Figure 4, that the restrictions placed on the pallet being removed from the storage area are similar to those in Figure 1, except that, as in Figure 2, the pallet does not have to be completely withdrawn from the storage area before the fork truck and pallet are free to rotate. This is accomplished by placing the pallet at an angle to the aisle. The greater the angle, the

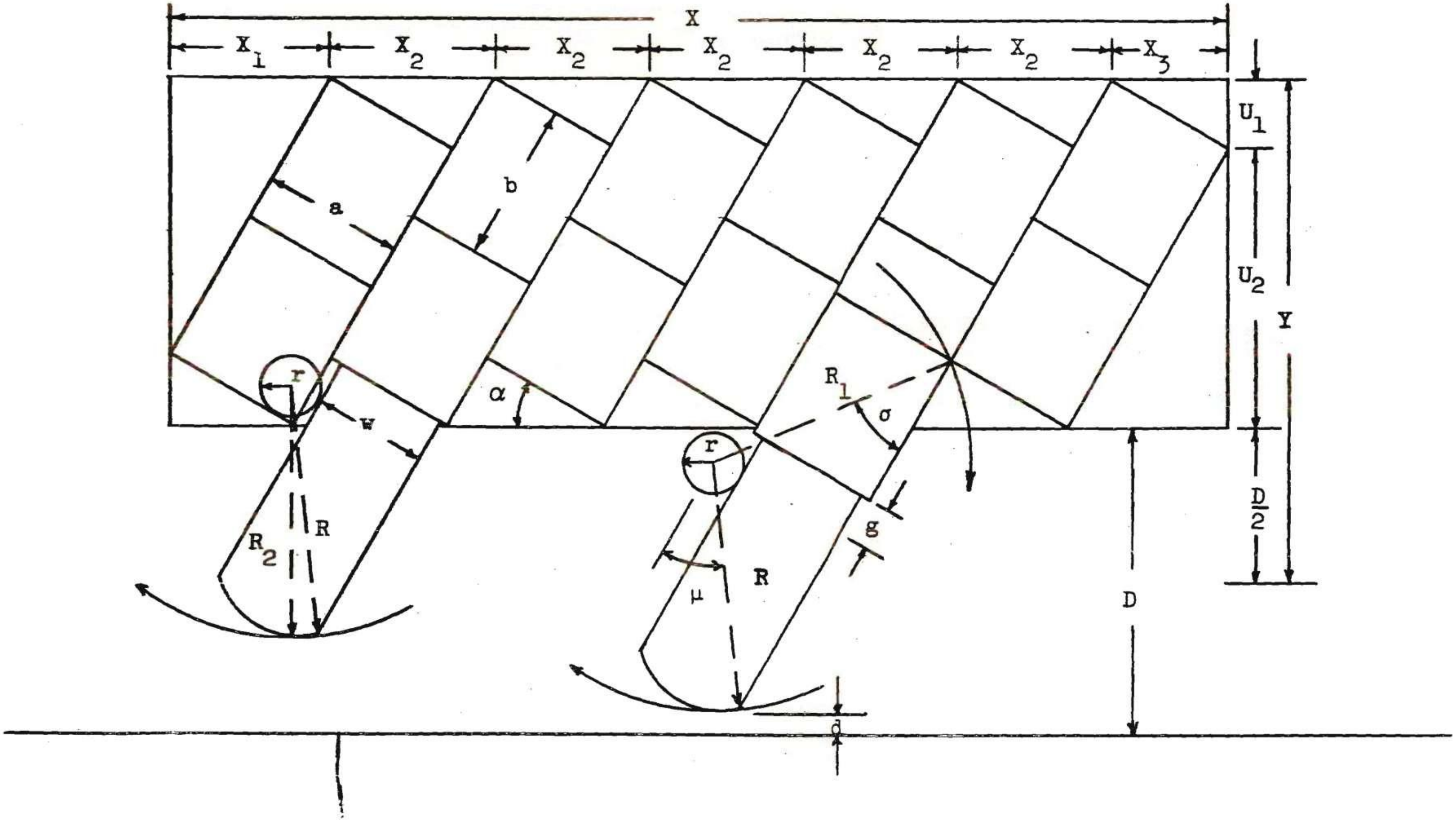

Figure 4. Angle Placement With No Clearance Between Pallets.

shorter the rearward movement of the fork truck before the end of the pallet clears the adjacent pallet and is free to rotate. For any given size of pallet there is an angle beyond which the pallet can not be rotated, since, beyond this point one pallet will mask entry into the face of the adjacent pallet. This restraint has been considered in the solution of the equation and is so noted in Table 3a in the next chapter.

With these facts in mind, the equations for aisle width for Case III can be written. The equations are shown as follows:

$$D = R + d + g \cos \alpha + b \cos \alpha - 2a \sin \alpha$$

$$\text{valid for } \mu \geq \alpha$$

and

$$D = R \cos (\alpha - \mu) + d + g \cos \alpha + b \cos \alpha - 2a \sin \alpha \tag{20}$$

$$\text{valid for } \mu \leq \alpha$$

where α is the angle of placement and μ is a function of the fork truck as given in equation (15). Reference to the table in the Appendix indicates that μ is always greater than 30° for the fork trucks studied.

Having developed equations for the aisle width D, in terms of known parameters, the expression for the efficiency of utilization of floor space is written as follows:

$$\text{AREA UTILIZED} = a\, b\, (N_w N_d + n)$$

$$\text{TOTAL AREA} = \left[a \cos \alpha + \frac{a(N_w - 1)}{\cos \alpha} + N_d\, n \sin \alpha \right] \left[a \sin \alpha + N_d b \cos \alpha + \frac{1}{2} (R + d + g \cos \alpha + b \cos \alpha - 2a \sin \alpha) \right]$$

which simplifies to

$$\text{TOTAL AREA} = \left[a \cos \alpha + \frac{(N_w - 1)}{\cos \alpha} + N_d b \sin \alpha \right]$$

$$\cdot \frac{1}{2} \left[\cos \alpha (2N_d\, b + g + d) + R + D \right]$$

$$= \frac{1}{2} [X] [Y]$$

The following expression is then given as the efficiency

$$E = \frac{200\, a\, b\, (N_w N_d + n)}{(X)\,(Y)} \,\% \, . \tag{22}$$

Examination of X and Y in equation (22) indicates that as α is increased from zero, X increases and Y decreases; however, the product XY increases monotonically in α thus, E is a maximum when $\alpha = 0$, and E decreases monotonically as α increases.

CHAPTER III

SOLUTION OF THE EQUATIONS

Examination of each of the efficiency equations indicated that the optimum values of the independent design parameters α and c were $\alpha = c = 0$, i.e., the optimum values occurred on the boundary of the range of interest for each of these variables. However, the greatest utility of these equations can be derived by showing the effect of variations in the various parameters on efficiency, as one departs from the optimum level of the various parameters studied. In this way, maximum efficiency designs, with constraints on the aisle width to accommodate existing warehouse structure, can be obtained. For this reason, an extensive set of tables has been prepared for each of the three cases studied in this thesis. In these tables, each of the parameters is varied, in suitable increments, over a range which should cover the region of interest to the practicing engineer. A table of corresponding aisle widths is also given.

As an example of how the solutions were obtained, the details of Case I, $c = 0$, $\alpha = 0$, are presented. This Case covered three sizes of pallet and two sizes of fork lift truck. The range of the pertinent variables evaluated are as follows:

$$b = 48'', 60'', 84''$$

$$N_d = 1, 2, 3, ----, 10$$

$$K = 80'', 110''$$

The sum of the variables g, R, and d, is denoted by K. The values for g and R were obtained from specifications published by the manufacturer. The value for d, which is warehouse practice, has been set at 6" and will remain constant throughout the calculations for all the Tables. To illustrate the mechanics of the computation, let us select a pallet with length, b = 48"; a fork truck with K = 80"; and depth of pallets, $N_d = 1$. Using equation (2), we immediately obtain

$$E = \frac{200\ (1)\ 48}{[2(1) + 1]\ 48 + 80} = 42.9\% \ .$$

Since most layout work is performed on existing buildings, the aisle width is of considerable importance along with the respective floor space efficiencies. Therefore, in this case, use equation (1) for the aisle width. Then

$$D = 48 + 13 + 61 + 6 = 128" \ .$$

For the convenience of the reader, a summary of the efficiencies for the Case I c = 0, $\alpha = 0$, are listed in Table 1a, and aisle widths in Table 4. In similar manner, appropriate tables were prepared for the other cases studied. In each Case, a representative range of fork truck and pallet size which might be encountered in industry were studied.

During the investigation of fork truck specifications, it was found that the dimension which varied most from one manufacturer to another was the turning radius of the rear of the fork truck R. The distance from the face of the forks to the center of the fork truck front axle g varied to some extent but not to the point where it could be considered significant.

The variation of this parameter was from 10 to 15 inches and did not materially affect the computations. The tabulation of the sum of the variables g, R, and d, which is denoted by K for the wide range of fork lift trucks manufactured, allowed for a grouping of the K values in such a manner that the average value of K for each size fork lift truck (i.e., 2000 lb., 3000 lb., 4000 lb., and other sizes studied) varied only slightly for the significant parameters g and R. The values selected for K therefore represent the average for each size group. The extreme values for K are 75" and 110", which the author found adequate to handle the pallet range studied.

Each manufacturer has rated the lifting capacity of specific fork lift trucks and shows these capacities on the specification sheet for each truck. These ratings should be taken into consideration when using the tables for the various size pallets since an increase or decrease in size of pallet will change the capacity of the fork lift truck. Usually the standard rating is shown in graph form, indicating the load length in inches and the capacity in pounds. A fork lift truck rated as 3000 lb. with 48" long load, equivalent to the load center 24" out from the heel of the forks, is also rated as 2000 lb. with 84" long load and load center 42" out from the heel of the forks. Other load and load length combinations within the capacity of this truck are shown in the manufacturer's specification sheet.

Table 1a. Floor Space Utilization Efficiency in Per Cent for Case I $\alpha = 0$, $c = 0$.

N_d	b = 84" K = 80"	b = 84" K = 110"	b = 60" K = 80"	b = 60" K = 110"	b = 48" K = 80"	b = 48" K = 110"
1	50.6	46.4	46.1	41.4	42.9	37.8
2	67.2	63.4	63.2	58.5	60.0	54.9
3	75.4	72.2	72.0	67.9	69.2	64.6
4	80.4	77.6	77.4	73.8	75.0	70.8
5	83.7	81.2	81.1	77.9	78.9	75.2
6	86.0	83.9	83.7	80.9	81.8	78.5
7	87.8	85.8	85.7	83.2	84.0	81.0
8	89.1	87.4	87.3	85.0	85.7	82.9
9	90.2	88.6	88.5	86.4	87.1	84.5
10	91.1	89.6	89.5	87.6	88.2	85.9

Table 1b. Loss in Efficiency Expressed as a Percentage of the b = 84", K = 80" Efficiency for Case I $\alpha = 0$, $c = 0$.

	b = 84"		b = 60"		b = 48"	
1	50.6	8.3	8.9	18.2	15.2	25.3
2	67.2	5.6	5.9	13.0	10.7	18.3
3	75.4	4.2	4.5	9.9	8.2	14.3
4	80.4	3.5	3.7	8.2	6.7	11.9
5	83.7	3.0	3.1	6.9	5.7	10.2
6	86.0	2.4	2.7	5.9	4.9	8.7
7	87.8	2.3	2.4	5.2	4.3	7.7
8	89.1	1.9	2.0	4.6	3.8	7.0
9	90.2	1.8	1.9	4.2	3.4	6.3
10	91.1	1.6	1.8	3.8	3.2	5.7

Variation between values of b and K is very nearly linear for Case I only.

Table 2a. Floor Space Utilization Efficiency in Per Cent for Case II $\alpha = 0$, $c \neq 0$.

		b x a = 36" x 40"				b x a = 40" x 40"				b x a = 48" x 40"			
K	N_d/c	0"	5"	10"	21.2"	0"	5"	10"	24.4"	0"	5"	10"	31.1"
	1	39.3	39.1	38.9	38.3	41.0	40.5	40.3	39.6	43.8	42.8	42.3	40.4
	2	56.5	55.2	54.2	51.4	58.2	56.6	55.5	52.2	61.0	58.9	57.3	51.8
	3	66.1	64.0	62.3	58.1	67.6	65.3	63.4	58.4	70.1	67.3	65.0	57.1
75"	4	72.2	69.6	67.3	62.1	73.6	70.7	68.3	62.1	75.7	72.4	69.7	60.2
	5	76.4	73.4	70.7	64.8	77.7	74.4	71.6	64.5	79.6	75.9	72.8	62.3
	7	82.0	78.3	75.0	68.1	83.0	79.1	75.8	67.6	84.5	80.4	76.8	64.8
	10	86.6	82.5	78.7	70.9	87.4	83.1	79.3	70.1	88.6	84.1	80.1	66.8
K	N_d/c	0"	5"	10"	23.0"	0"	5"	10"	26.2"	0"	5"	10"	33.3"
	1	38.3	37.8	37.6	36.8	40.0	39.2	38.9	37.7	42.9	41.7	41.0	38.5
	2	55.4	53.9	52.8	49.9	57.1	55.4	54.1	50.2	60.0	57.8	56.1	49.8
80"	3	65.1	62.9	61.1	56.7	66.7	64.2	62.2	56.5	69.2	66.3	64.0	55.3
	4	71.3	68.6	66.3	60.8	72.7	69.7	67.3	60.3	75.0	71.6	68.8	58.4
	5	75.6	72.5	69.8	63.5	76.9	73.5	70.9	62.8	78.9	75.2	72.1	60.5
	7	81.3	77.6	74.4	67.0	82.4	78.4	75.1	65.9	84.0	79.8	76.2	63.1
	10	86.1	82.0	78.2	69.9	87.0	82.6	78.8	68.5	88.2	83.6	79.6	65.2
K	N_d/c	0"	5"	10"	23.0"	0"	5"	10"	26.2"	0"	5"	10"	33.3"
	1	37.3	36.7	36.5	35.6	39.0	38.2	37.9	36.5	41.9	40.7	40.0	37.5
	2	54.3	52.8	51.8	48.8	56.1	54.4	53.1	49.2	59.1	56.9	55.2	48.9
	3	64.1	61.9	60.1	55.7	65.8	63.3	61.3	55.6	68.4	65.5	63.2	54.5
85"	4	70.4	67.7	65.4	60.0	71.9	68.9	66.5	59.5	74.3	70.9	68.1	57.8
	5	74.8	71.7	69.1	62.8	76.2	72.8	80.0	62.1	78.3	74.6	71.5	60.0
	7	80.6	76.9	73.8	66.5	81.8	77.8	74.5	65.4	83.5	79.3	75.7	62.7
	10	85.6	81.4	77.7	69.5	86.5	82.1	78.3	68.1	87.8	83.2	79.2	64.9
K	N_d/c	0"	5"	10"	21.3"	0"	5"	10"	24.5"	0"	5"	10"	27.0"
	1	36.4	36.1	35.8	35.1	38.1	37.4	37.1	36.0	41.0	40.1	39.1	38.6
	2	53.3	52.1	51.0	48.6	55.2	53.6	52.4	49.0	58.2	56.2	54.3	50.9
	3	63.2	61.2	59.5	55.7	64.9	62.6	60.7	55.7	67.6	64.9	62.4	57.0
90"	4	69.6	67.1	64.8	60.2	71.1	68.3	65.9	59.8	73.6	70.4	67.4	60.6
	5	74.1	71.2	68.5	63.2	75.5	72.2	69.5	62.5	77.7	74.1	70.8	63.0
	7	80.0	76.5	73.3	67.0	81.2	77.4	74.1	66.0	83.0	78.9	75.1	65.9
	10	85.1	81.1	77.4	70.2	86.0	81.7	78.0	68.8	87.4	82.9	78.8	68.4

Table 2a (Continued)

K	N_d/c	b x a = 36" x 40"				b x a = 40" x 40"				b x a = 48" x 40"			
K	N_d/c	0"	5"	10"	21.3"	0"	5"	10"	24.5"	0"	5"	10"	27.0"
	1	35.5	35.1	34.8	34.0	37.2	36.5	36.2	35.0	40.2	39.2	38.2	37.6
	2	52.4	51.1	50.0	47.6	54.2	52.6	51.4	48.0	57.3	55.4	53.4	50.0
	3	62.2	60.3	58.6	54.9	64.0	61.7	59.8	54.8	66.8	64.2	61.6	56.2
95"	4	68.7	66.2	64.0	59.4	70.3	67.5	65.2	59.0	72.9	69.7	66.7	59.9
	5	73.3	70.4	67.8	62.5	74.8	71.6	68.8	61.9	77.0	73.5	70.2	62.3
	7	79.4	75.9	72.7	66.4	80.6	76.8	73.6	65.5	82.5	78.4	74.7	65.5
	10	84.6	80.5	76.9	69.8	85.6	81.3	77.6	68.4	87.0	82.6	78.4	68.0
K	N_d/c	0"	5"	10"	20.4"	0"	5"	10"	22.5"	0"	5"	10"	28.8"
	1	34.6	34.2	34.0	33.4	36.3	35.8	35.4	34.7	39.3	38.5	37.8	36.0
	2	51.4	50.2	49.2	47.0	53.3	51.8	50.7	48.0	56.5	54.6	53.0	48.3
	3	61.4	59.4	58.5	54.5	63.2	61.0	59.1	55.1	66.1	63.5	61.3	54.6
100"	4	68.1	65.5	63.3	59.1	69.6	66.9	64.5	59.5	72.2	69.1	66.4	58.3
	5	72.6	69.7	67.2	62.4	74.1	71.0	68.3	62.4	76.4	73.0	70.0	60.9
	7	78.8	75.3	72.2	66.5	80.0	76.3	73.1	66.2	82.0	78.0	74.5	64.0
	10	84.1	80.1	76.5	70.0	85.1	80.9	77.2	69.4	86.6	82.2	78.3	66.6
K	N_d/c	0"	5"	10"	20.4"	0"	5"	10"	22.5"	0"	5"	10"	28.8"
	1	33.0	32.6	32.3	31.5	34.8	34.2	33.7	32.9	37.8	36.9	36.2	34.3
	2	49.7	48.4	47.4	45.2	51.6	50.1	48.9	46.3	54.9	53.0	51.4	46.7
	3	59.7	57.7	56.1	52.8	61.5	59.4	57.5	53.5	64.6	62.1	59.8	53.2
110"	4	66.4	63.9	61.8	57.7	68.1	65.4	63.1	58.1	70.8	67.8	65.2	57.2
	5	71.1	68.3	65.8	61.0	72.7	69.7	67.0	61.2	75.2	71.8	68.8	59.8
	7	77.5	74.1	71.1	65.4	78.9	75.2	72.1	65.2	81.0	77.1	73.6	63.2
	10	83.1	79.2	75.6	69.1	84.2	80.1	76.4	68.6	85.9	81.5	77.6	66.0

Table 2b. Loss in Efficiency Expressed as a Percentage of the c = 0 Efficiency for Case II $\alpha = 0$, $c \neq 0$.

		b x a = 36" x 40"				b x a = 40" x 40"				b x a = 48" x 40"			
K	N_d/c	0"	5"	10"	21.2"	0"	5"	10"	24.4"	0"	5"	10"	31.1"
	1	39.3	.5	1.0	2.6	41.0	1.2	1.7	3.4	43.8	2.3	3.4	7.8
	2	56.5	2.3	4.1	9.0	58.2	2.8	4.6	10.3	61.0	3.5	6.1	15.1
75"	3	66.1	3.2	5.8	12.2	67.6	3.4	6.2	13.6	70.0	4.0	7.3	18.6
	4	72.2	3.6	6.8	14.0	73.6	4.0	7.2	15.6	75.7	4.4	7.9	20.5
	5	82.0	4.5	8.5	15.2	83.0	4.7	8.7	18.6	84.5	4.9	9.1	23.3
	7	82.0	4.5	8.5	17.0	83.0	4.7	8.7	18.6	84.5	4.9	9.1	23.3
	10	86.6	4.7	9.1	18.1	87.4	4.9	9.3	19.8	88.6	5.1	9.6	24.6
K	N_d/c	0"	5"	10"	23.0	0"	5"	10"	26.2"	0"	5"	10"	33.3"
	1	38.3	1.3	1.8	3.9	40.0	2.0	2.8	5.8	42.9	2.8	4.4	10.3
	2	55.4	2.7	4.7	9.9	57.1	3.0	5.3	12.1	60.0	3.7	6.5	17.0
	3	65.1	3.4	6.2	12.9	66.7	3.8	6.8	15.3	69.2	4.2	7.5	20.1
80"	4	71.3	3.8	7.0	14.7	72.7	4.1	7.4	17.1	75.0	4.5	8.3	22.1
	5	75.6	4.1	7.7	16.0	76.9	4.4	8.1	18.3	78.9	4.7	8.6	23.3
	7	81.3	4.6	8.5	17.6	82.4	4.9	8.9	20.0	84.0	5.0	9.3	24.9
	10	86.1	4.8	9.2	18.8	87.0	5.1	9.4	21.3	88.2	5.2	9.8	26.1
K	N_d/c	0"	5"	10"	23.0"	0"	5"	10"	26.2"	0"	5"	10"	33.3"
	1	37.3	1.6	2.2	4.6	39.0	2.1	2.8	6.4	41.9	2.9	4.5	10.5
	2	54.3	2.8	4.6	10.1	56.1	3.0	5.4	12.3	59.1	3.7	6.6	17.3
	3	64.1	3.4	6.3	13.1	65.8	4.1	6.8	15.5	68.4	4.2	7.6	20.3
85"	4	70.4	3.8	7.1	14.8	71.9	4.2	7.5	17.3	74.3	4.6	8.4	22.2
	5	74.8	4.2	7.6	16.1	76.2	4.5	8.1	18.5	78.3	4.7	8.7	23.4
	7	80.6	4.6	8.4	17.5	81.8	4.9	8.9	20.1	83.5	5.0	9.4	24.9
	10	85.6	4.9	9.2	18.8	86.5	5.1	9.5	21.3	87.8	5.2	9.8	26.1
K	N_d/c	0"	5"	10"	21.3"	0"	5"	10"	24.5"	0"	5"	10"	27.0"
	1	36.4	.8	1.7	3.6	38.1	1.8	2.6	5.5	41.0	2.2	4.6	5.9
	2	53.3	2.3	4.3	8.8	55.2	2.9	5.1	11.2	58.2	3.4	6.7	12.6
	3	63.2	3.2	5.9	11.9	64.9	3.6	6.5	14.2	67.6	4.0	7.7	15.7
90"	4	69.6	3.6	6.9	13.5	71.1	3.9	7.3	15.9	73.6	4.4	8.4	17.7
	5	74.1	3.9	7.6	14.7	75.5	4.4	8.0	17.2	77.7	4.6	8.9	18.9
	7	80.0	4.4	8.4	16.3	81.2	4.7	8.8	18.7	83.0	4.9	9.5	20.6
	10	85.1	4.7	9.1	17.5	86.0	5.0	9.3	20.0	87.4	5.2	9.8	21.7

Table 2b (Continued)

		b x a = 36" x 40"				b x a = 40" x 40"				b x a = 48" x 40"			
K	N_d/c	0"	5"	10"	21.3"	0"	5"	10"	24.5"	0"	5"	10"	27.0"
	1	35.5	1.1	2.0	4.2	37.2	1.9	2.7	5.9	40.2	2.5	5.0	6.5
	2	52.4	2.5	4.6	9.2	54.2	3.0	5.2	11.4	57.3	3.3	6.8	12.7
	3	62.2	3.1	5.8	11.7	64.0	3.6	6.6	14.4	66.8	3.9	7.8	15.9
95"	4	68.7	3.6	6.9	13.5	70.3	4.0	7.3	16.1	72.9	4.4	8.5	17.8
	5	73.3	4.0	7.5	14.7	74.8	4.3	8.0	17.3	77.0	4.6	8.8	19.1
	7	79.4	4.4	8.4	16.4	80.6	4.7	8.7	18.7	82.5	5.0	9.5	20.6
	10	84.6	4.9	9.1	17.5	85.6	5.0	9.4	20.1	87.0	5.1	9.9	21.8
K	N_d/c	0"	5"	10"	20.4"	0"	5"	10"	22.5"	0"	5"	10"	28.8"
	1	34.6	1.2	1.7	3.5	36.3	1.4	2.5	4.4	39.3	2.0	3.8	8.4
	2	51.4	2.3	4.3	8.6	53.3	2.8	4.9	10.2	56.5	3.4	6.2	14.5
	3	61.4	3.3	4.7	11.2	63.2	3.5	6.5	12.8	66.1	3.9	7.3	17.4
100"	4	68.1	3.8	7.1	13.2	69.6	3.9	7.3	14.5	72.2	4.3	8.0	19.3
	5	72.6	4.0	7.4	14.1	74.1	4.2	7.8	15.8	76.4	4.5	8.4	20.3
	7	78.8	4.5	8.4	15.6	80.0	4.6	8.6	17.3	82.0	4.9	9.2	22.0
	10	84.1	4.8	9.0	16.8	85.1	4.9	9.3	18.5	86.6	5.1	9.6	23.1
K	N_d/c	0"	5"	10"	20.4"	0"	5"	10"	22.5"	0"	5"	10"	28.8"
	1	33.0	1.2	2.1	4.6	34.8	1.7	3.2	5.5	37.8	2.4	4.2	9.3
	2	49.7	2.6	4.6	9.1	51.6	2.9	5.2	10.3	54.9	3.5	6.4	14.9
	3	49.7	3.4	6.0	11.6	61.5	3.4	6.5	13.0	64.6	3.9	7.4	17.7
110"	4	66.4	3.8	6.9	13.1	68.1	4.0	8.4	14.7	70.8	4.2	7.9	19.2
	5	71.1	3.9	7.5	14.2	72.7	4.1	7.9	15.8	75.2	4.5	8.5	20.5
	7	77.5	4.4	8.3	15.6	78.9	4.7	8.6	17.4	81.0	4.8	9.1	22.0
	10	83.1	4.7	9.0	16.9	84.2	4.9	9.3	18.5	85.9	5.1	9.7	23.2

Table 3a. Floor Space Utilization Efficiency in Per Cent for Case III $\alpha \neq 0$, $c = 0$, $N_w = 100$.

	b x a = 48" x 40", K = 80"						b x a = 48" x 40", K = 110"					
N_d/α	0°	10°	20°	30°	40°	50.2°	0°	10°	20°	30°	40°	50.2°
1	42.9	42.6	41.9	40.8	39.0	36.8	37.8	37.5	36.8	35.6	34.0	32.0
2	60.5	59.6	58.8	57.6	55.8	53.6	54.8	54.4	53.6	52.2	50.5	48.2
3	69.2	68.6	67.8	66.6	65.0	62.9	64.6	64.0	63.1	61.7	60.0	57.9
4	75.0	74.3	73.3	72.1	70.6	68.8	70.8	70.1	69.1	67.7	66.1	64.2
5	78.9	78.0	77.0	75.8	74.4	72.8	75.2	74.3	73.2	71.9	70.3	68.6
7	84.0	82.7	81.4	80.2	78.9	77.6	80.9	79.7	78.4	76.9	76.9	74.1
10	88.2	86.4	84.7	83.3	82.0	81.1	85.9	84.1	82.3	80.8	79.4	78.3

	b x a = 60" x 40", K = 80"						b x a = 60" x 40", K = 110"					
N_d/α	0°	10°	20°	30°	40°	56.3°	0°	10°	20°	30°	40°	56.3°
1	46.1	45.9	45.2	44.1	42.5	38.3	41.4	41.1	40.4	39.2	37.7	34.2
2	63.2	67.7	61.9	60.8	59.2	55.0	58.5	58.0	57.2	55.9	54.3	50.6
3	72.0	71.3	70.4	69.3	67.8	64.2	67.9	67.2	66.3	65.0	63.4	60.1
4	77.4	76.5	75.5	74.3	73.0	69.9	73.8	73.0	71.9	70.6	69.1	66.2
5	81.1	80.0	78.8	77.6	76.3	73.6	77.9	76.8	75.6	74.2	72.8	70.3
7	85.7	84.1	82.7	81.3	80.1	78.1	83.2	81.6	80.1	78.6	77.3	75.4
10	89.5	87.3	85.3	83.6	82.4	81.1	87.6	85.3	83.3	81.6	80.3	79.0

	b x a = 84" x 40", K = 80"						b x a = 84" x 40", K = 110"					
N_d/α	0°	10°	20°	30°	40°	65.4°	0°	10°	20°	30°	40°	65.4°
1	50.6	50.3	49.6	48.7	47.3	44.8	46.4	46.1	45.4	44.3	42.9	40.8
2	67.2	66.7	65.9	64.9	63.6	61.4	63.4	62.8	62.0	60.9	59.5	57.6
3	75.4	74.7	73.8	72.8	71.6	69.8	72.2	71.5	70.5	69.4	68.0	66.4
4	80.4	79.4	78.4	77.3	76.2	74.8	77.6	76.6	75.6	74.4	73.1	71.8
5	83.7	82.5	81.3	80.2	79.1	77.9	81.2	80.0	78.8	77.6	76.4	75.3
7	87.8	86.1	84.6	83.3	82.3	81.5	85.8	84.2	82.6	81.2	80.1	79.4
10	91.1	88.7	86.7	85.1	84.0	83.7	89.6	78.3	85.2	83.5	82.4	82.1

Table 3b. Loss in Efficiency Expressed as a Percentage of the $\alpha = 0$ Efficiency For Case III $\alpha \neq 0$, $c = 0$, $N_w = 100$.

	b x a = 48" x 40", K = 80"						b x a = 48" x 40", K = 110"					
N_d/α	0°	10°	20°	30°	40°	50.2°	0°	10°	20°	30°	40°	50.2°
1	42.9	.7	2.3	4.9	9.1	14.2	37.8	.8	2.7	5.8	10.1	15.4
2	60.0	.7	2.0	4.0	7.0	10.7	54.8	.7	2.2	4.8	7.9	12.1
3	69.2	.9	2.0	3.8	6.1	9.1	64.6	.9	2.3	4.5	7.1	10.4
4	75.0	.9	2.3	3.9	5.9	8.3	70.8	1.0	2.4	4.4	6.6	9.3
5	78.9	1.2	2.4	3.9	5.7	7.7	75.2	1.2	2.7	4.4	6.5	8.8
7	84.0	1.6	3.1	4.5	6.1	7.6	80.9	1.5	3.1	5.0	5.0	8.4
10	88.2	2.1	4.0	5.6	7.0	8.1	85.9	2.1	4.2	5.9	7.6	8.9

	b x a = 60" x 40", K = 80"						b x a = 60" x 40", K = 110"					
N_d/α	0°	10°	20°	30°	40°	56.3°	0°	10°	20°	30°	40°	56.3°
1	46.1	.4	2.0	4.3	7.8	16.9	41.4	.7	2.4	5.3	8.9	17.4
2	63.2	.8	2.1	3.8	6.3	13.0	58.5	.9	2.2	4.4	7.2	13.5
3	72.0	1.0	2.2	3.8	5.8	10.8	67.9	1.0	2.4	4.3	6.6	11.5
4	77.4	1.2	2.5	4.0	5.7	9.7	73.8	1.1	2.6	4.3	6.4	10.3
5	81.1	1.4	2.8	4.3	5.9	9.3	77.9	1.4	3.0	4.7	6.6	9.8
7	85.7	1.9	3.5	5.1	6.5	8.9	83.2	1.9	3.7	5.5	7.1	9.4
10	89.5	2.5	4.7	6.6	7.9	9.4	87.6	2.6	4.9	6.8	8.3	9.8

	b x a = 84" x 40", K = 80"						b x a = 84" x 40", K = 110"					
N_d/α	0°	10°	20°	30°	40°	65.4°	0°	10°	20°	30°	40°	65.4°
1	50.6	.6	2.0	3.8	6.5	11.5	46.4	.7	2.2	4.5	7.6	12.1
2	67.2	.8	1.9	3.4	5.4	8.6	63.4	1.0	2.2	4.0	6.2	9.2
3	75.4	.9	2.1	3.5	5.0	7.4	72.2	1.0	2.4	3.9	5.8	8.0
4	80.4	1.3	2.5	3.9	5.2	7.0	77.6	1.3	2.6	4.1	5.8	7.5
5	83.7	1.4	2.9	4.2	5.5	6.9	81.2	1.5	3.0	4.4	5.9	7.3
7	87.8	1.9	3.7	5.1	6.3	7.2	85.8	1.9	3.7	5.4	6.7	7.5
10	91.1	2.6	4.8	6.6	7.8	8.1	89.6	2.6	4.9	6.8	8.0	8.4

Table 4. Aisle Width (Inches)

	Case I $\alpha = 0$, $c = 0$						
b x a/K	75"	80"	85"	90"	95"	100"	110"
48" x 40"	123	128	133	138	143	148	158
60" x 40"	135	140	145	150	155	160	170
84" x 40"	159	164	169	174	179	184	194

	Case II $\alpha = 0$, $c \neq 0$											
K/c	0"	5"	10"	Max	0"	5"	10"	Max	0"	5"	10"	Max
75"	111	101.5	91.6	75	115	106.0	96.4	75	123	114.9	105.7	75
80"	116	107.4	98.3	80	120	112.0	102.8	80	128	120.7	112.1	80
85"	121	112.4	103.3	85	125	117.0	107.8	85	133	125.7	117.1	85
90"	126	115.9	106.9	90	130	121.0	111.5	90	138	129.3	122.4	90
95"	131	120.9	111.9	95	135	126.0	116.5	95	143	134.3	127.4	95
100"	136	126.0	116.0	100	140	130.4	120.7	100	148	138.7	129.9	100
110"	146	136.0	126.0	110	150	140.4	130.7	110	158	148.7	139.9	110

	Case III $\alpha \neq 0$, $c = 0$							
b x a/α	10°	20°	30°	Max.	10°	20°	30°	Max.
48" x 40"	113.2	96.9	79.8	66.0	143.2	126.8	109.6	99.7
60" x 40"	125.0	108.2	90.2	64.9	155.0	138.1	120.0	101.0
84" x 40"	148.6	130.8	111.0	64.8	178.6	160.7	140.7	100.9

Note: Column headings for Case II from 0" to Max read from left to right as b x a equals 36" x 40", 40" x 40", and 48" x 40".

Column headings for Case III from 10° to Max read from left to right as K = 80" and K = 110".

CHAPTER IV

DISCUSSION OF RESULTS

In general we can conclude that as c and α increase the efficient use of warehouse space decreases. The optimum for these two parameters is when $c = 0$, $\alpha = 0$. It must be stated, however, that under actual operating conditions the parameter c can never equal zero since no fork lift operator is proficient enough to place or withdraw pallets from storage without a minimum clearance between them. This does not mean inefficiency is to be condoned in the performance of operators, for if this situation exists more warehouse space than is necessary will be used resulting in an increase in building cost. It is the responsibility of the engineer to insist that fork lift operators be sufficiently proficient in maneuvering the truck to allow the spacing of pallets in the storage area with a minimum clearance for maximum efficient use of the space alloted. For practical purposes, it is possible to allow up to 5" lateral clearance between pallets without an excessive loss in space efficiency.

It is evident that very little is to be gained in efficiency by placing pallets in storage deeper than $N_d = 5$. Investigation of the tables indicate that the increase in efficiency falls off sharply after the fifth pallet and continues to fall off the deeper the pallets are placed in the storage area. The first five pallets in depth produce an average increase in efficiency of 40% while the next five in depth, to a total of ten pallets in depth, produce an average additional increase in efficiency of only

10%. If a large number of different items are to be placed in the storage area, it would be more practical to limit the depth of storage to five pallets to take advantage of the maximum efficient use of the space for all items. One other factor affecting the use of the storage area is the length of time the item is to remain in the storage area before withdrawal. Any item which has a long storage cycle, before being completely depleted, decreases the efficient use of the space if placed in the storage area with a large N_d and, since the space occupied by the pallets cannot be used for other items, it will remain vacant for excessive periods of time. In cases such as this, it may be preferable to use multiple storage positions of smaller depth.

If the material to be placed on the pallet is bulky and lightweight, the engineer will do well to use as large a pallet as is practical. A 3000 lb. fork lift truck with a 24" load center also has a capacity of 2000 lb. with a load center of 32". The tables indicate that the larger the parameter b, the greater the efficiency of space utilization. It is also evident that the smaller the value of K, the greater is the efficiency. It can be concluded, from these facts, that the pallet should be as large as possible and the fork truck as small as the rated capacity will allow to lift the load.

In all instances where $\alpha = 0$, the efficiency is independent of the parameters a and N_w. This indicates that the pallet may be as wide or as narrow as is expedient, subject to the constraint that $a \leq D - d$, and not affect the efficiency. This fact can be of great value if a large number of items are to be stored in an area with a limited aisle length. It may be desirable to make the pallets narrow in order to insure that each item has a facing on the aisle. This latitude is of particular value since most layout work is undertaken on existing buildings and the engineer is limited to work within the dimensions of the building and column spacing.

When $\alpha \neq 0$, all the parameters have an effect on the efficiency although some affect it more than others. The larger the value of α, the greater effect the parameter N_d has on efficiency. If α is maximum for a given pallet, we see immediately that the loss in efficiency in all the tables is approximately 10%, when $N_d = 10$. If added to this is the loss sustained when c is maximum, the loss in efficiency is so great the condition can not be considered under any circumstance.

The value of the tables is that they allow the engineer to make this choice without proceeding with time consuming calculations to arrive at a decision as to pallet size, fork truck specifications, angle of placement, and lateral clearance between pallets. Since this study covers the practical range of warehouse pallet and fork truck equipment, it should be possible to make most decisions regarding the layout by selecting the values in the tables that allow maximum efficiency for the area under study. However, the engineer must remember that no consideration has been given to the effect pallet placement may have on labor. This factor should be weighed carefully before deciding which layout will produce the maximum warehouse operating efficiency.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The equations developed in this study should provide the Industrial Engineer with a tool by which can be measured the effectiveness of various equipment on cost and the productive use of warehouse space, either in existing buildings or in the development of requirements for construction of new ones. There should also be dispelled from the mind of the reader conflicts which have existed through manufacturers' claims which could not be measured effectively but which may now be evaluated properly with respect to the degree of efficiency each unit can produce.

It is evident that the practicing engineer should take into consideration pallet size and fork truck capabilities in conjunction with each other rather than make decisions regarding the types to be used separately. The reader will note in the tables that there are many combinations which will produce the same per cent efficiency, thus allowing numerous equivalent alternatives in the layout. This makes it possible to develop alternative layouts employing different equipment and to evaluate each on the basis of other factors. Since the several combinations produce the same efficiencies, we may gain some advantage in terms of equipment cost which would not be apparent if developed in any other manner.

Because of the approach that was employed, it was necessary to limit the range of pallet size and fork truck specifications. No consideration was given to the effect on fork truck lifting capacity when the pallet length was increased. It would seem practical to investigate the loss in efficiency by increasing the counterbalance of the fork truck as against the increase in efficiency by using the larger size pallets. The increase in counterbalance would insure the same capacity when using a pallet of a size larger than the rated capacity of the fork truck and at the same time increase the efficiency. Many fork truck manufacturers accomplish this by using the chassis of a smaller fork truck with the counterbalance and mast of a larger truck.

No consideration has been given to the length of time a pallet remains in storage and the effect this might have on the depth of storing the pallets. This decision would have to be made based on the movement of the items to be stored and would probably vary considerably from one item to the next. It should be pointed out, however, that careful consideration must be given to this point when making the layout. Items placed in storage for longer than normal periods and withdrawn over extended periods of time leave areas unoccupied which could be put to more profitable use.

Probably the most important single factor which has not been considered in this study is the cost of labor to perform under the various states of the study. The author recommends that before reaching a final decision, the engineer study the effect on cost of labor and building cost under each of the alternatives of interest, for the cost of one may cancel the savings anticipated in the other. It is known that as α increase the cost of labor decreases. How much effect this may have on the cost of labor can only be surmised at this point but the effect may be sufficiently large in terms of cost to outweigh the loss in floor space efficiency. This study shows that in some cases, optimum efficiency is obtained when $\alpha = 0$; it may be in some cases, that when the cost of labor is taken into consideration, the optimum efficiency is obtained when $\alpha \neq 0$.

A P P E N D I X

APPENDIX

Fork Truck and Pallet Specifications

Fork Truck						Pallet					
						36" x 40"		40" x 40"		48" x 40"	
K	W	r	g	R	μ	R_1	σ	R_1	σ	R_1	σ
75"	34	0	10	59	35.2	59.0	38.8	62.2	36.5	68.8	32.5
80"	34	0	13	61	33.9	61.4	37.1	64.6	34.9	71.4	31.2
85"	34	0	13	66	31.0	61.4	37.1	64.6	34.9	71.4	31.2
90"	38	6	15	69	39.6	68.0	41.4	71.1	39.3	77.4	35.5
100"	38	9	15	79	36.5	70.0	43.3	73.0	41.1	79.2	37.3
110"	38	9	15	89	31.9	70.0	43.3	73.0	41.1	79.2	37.3

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